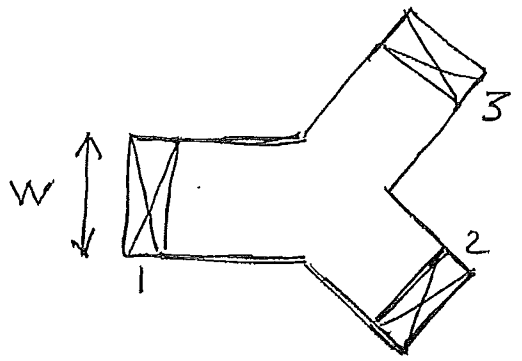
II

Exam Mesoscopic Physics 17-6-2015 9:00-12:00 5173.0055

Write each answer and your name and study number on a separate sheet. Indicate for every answer how it is obtained! There are 5 questions.

1) (total 25pts) Consider a two-dimensional electron gas with three contacts 1, 2 and 3. The contacts can each transmit N channels and have a width W. A current is injected in contact 1 and taken out at contact 3. A voltage is measured between contacts 1 and 2



a) Describe how the Landauer-Buttiker formalism is applied for each of the three contacts. Write down the equations of the currents in each of the contacts in terms of reflection and transmission coefficients, the number of channels N, and the electrochemical potentials of the contacts 1, 2, and 3. (6pts)
b) Give an estimate for all reflection and transmission coefficients for B=0. (4pts)
c) Derive an expression for the measured voltage in terms of the transmission and reflection coefficients. (6pts)

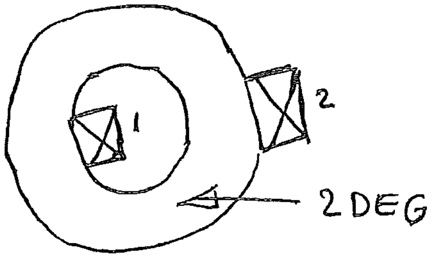
Now a perpendicular magnetic field B is applied (the corresponding cyclotron motion of the electrons is clockwise)

d) Describe how the reflection and transmission coefficients are changed. Do this for two regimes: $2\,l_c < W$ and $2\,l_c > W$). (3pts)
e) Describe how the application of this magnetic field changes the voltage calculated under c) (3pts)
f) Now the direction of the magnetic field is reversed. Does this change the measured voltage? Why (or why not)? (3pts)

2) (total 18pts) In a standard quantum Hall bar structure, Shubnikov de Haas (SdH) oscillations are observed in the longitudinal resistance when the magnetic field is changed (see lecture notes Introduction to Mesoscopic Physics)

a) At high magnetic fields the longitudinal resistance becomes zero at the same values of magnetic field where the quantum Hall effect shows plateaus. Why? (3pts)
b) Argue why the resistance peaks in the SdH oscillations increase with magnetic field. (3pts)

The figure illustrates a so-called Corbino geometry. Here one contact is on the inside of a ring shaped two-dimensional electron gas, and another contact is on the outside.



c) Draw the edge channels for the case of two occupied Landau Levels. (3pts)

d) Calculate/argue what the conductance is under quantum Hall conditions, when the Fermi energy is located in between consecutive Landau levels. (3pts)

e) Make a schematic plot of the conductance as when the magnetic field is swept and the number of occupied Landau levels is reduced. (3pts)

f) The behavior of the conductance of Corbino geometry is different from the longitudinal or Hall resistance of a conventional Hall bar geometry. Why? (3pts)

3) (total 20pts) Compare the two-dimensional properties of graphene and a semiconductor two-dimensional electron gas formed in a GaAs/AlGaAs heterostructure.

a) Give an estimate in what range of Fermi energies the electron states are strictly two dimensional. (5pts)

Consider now the strictly 2D regime.

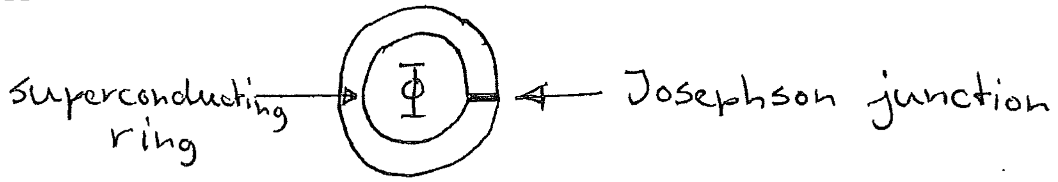
b) The E(k) relation for the semiconductor two-dimensional electron gas is given by $E(k) = \hbar^2 k^2/2m^*$, where m^* is the effective mass. Calculate the two-dimensional density of states, and show that is constant as function of energy. (5pts)

c) The E(k) relation for graphene (close to the K' and K points) is given by: $E(k)= \hbar v_F k$. Using a similar formalism as in b) calculate the two-dimensional density of states, and show that it increases linearly with energy. (5pts)

d) Compare the motion and scattering of two-dimensional electrons in graphene and a semiconductor. Can you describe some differences and similarities? (5pts)

4) (total 15 pts) Compare the circulating persistent/supercurrent in two systems:

The first system is a superconducting loop, interrupted by a Josephson Junction with critical current I_c. (see figure) The critical current is much smaller than the maximum supercurrent that the loop can support.



a) Calculate the persistent/super current as a function of the enclosed flux Φ, and draw it as a function of flux. (5pts)

Compare this with the persistent current carried by a one-dimensional (ballistic) wire which is made in the form of a ring with radius R. The effective mass is m^*

b) Describe how the energy levels depend on the magnetic flux. Draw the energy levels as a function of magnetic flux. Draw the persistent current as a function of magnetic flux. (6pts)

c) Compare the maximum persistent/supercurrent for the superconducting and normal ring. How does it depend on the diameter of the ring in both cases ? How does the temperature change the persistent/supercurrent? (4pts)

5) (total 23 pts) A three dimensional metallic system has a Fermi energy E_F and the carriers have an effective mass m*. The thickness D of the system (in the z-direction) is reduced.

a) Calculate the thickness D where the system becomes two-dimensional. Assume that the electrons are confined by a "hard wall"potential. (6pts)

b) For this thickness D plot the (total) density of states as a function of energy. (6pts)

Now electrons are confined in the z-direction by a (smooth) confinement potential $V= - \frac{1}{2} m^* \omega^2 z^2$. This potential corresponds to a harmonic oscillator potential with frequency ω.

c) Calculate what the value of ω should be for this system to be two-dimensional. (6pts)

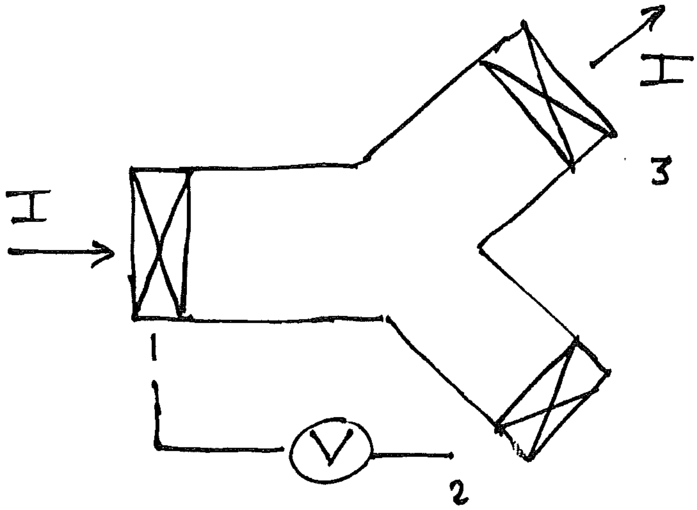
d) In the case when the two-dimensional regime is reached do you think that the shape of the confinement potential in the z-direction matters ? Describe why or why not. (5pts)

Mesoscopic Physics 17-6-2015 A

III

1)

a)



$$I = I_1 = N\frac{2e}{h}\left[\mu_1(1-R_{11}) - \mu_2 T_{21} - \mu_3 T_{31}\right] \qquad (1)$$

$$-I = I_3 = N\frac{2e}{h}\left[\mu_3(1-R_{33}) - \mu_2 T_{23} - \mu_1 T_{13}\right] \qquad (2)$$

$$0 = I_2 = N\frac{2e}{h}\left[\mu_2(1-R_{22}) - \mu_1 T_{12} - \mu_3 T_{32}\right] = 0 \qquad (3)$$

$\boxed{6pt}$

b) $R_{11} = 0 \quad T_{12} = 0.5 \quad T_{13} = 0.5$

$R_{22} \approx 0.1 \quad T_{21} \approx 0.5 \quad T_{23} \approx 0.4$

$R_{33} \approx 0.1 \quad T_{31} \approx 0.5 \quad T_{32} \approx 0.4$

note: $T_{\alpha\beta} = T_{\beta\alpha}$ in absence of magnetic field

$\boxed{4pt}$

c) (3) $\mu_2(1-R_{22}) - \mu_1 T_{12} = 0 \qquad \mu_2 = \frac{T_{12}}{1-R_{22}}\mu_1$

(1) $\mu_1\left\{(1-R_{11}) - \frac{T_{12}T_{21}}{1-R_{22}}\right\} = \frac{-I}{\frac{2e}{h}N}$

(2) $\mu_1\left\{-T_{13} - \frac{T_{23}T_{12}}{1-R_{22}}\right\} = \frac{I}{\frac{2e}{h}N}$

calculate $\frac{\mu_2 - \mu_3}{I}$ from (1) (2) and (3)

$\boxed{6pt}$

(B)

d) assume clockwise circlotron motion

T_{12}, T_{23}, T_{31} increase
T_{21}, T_{32}, T_{13} decrease
R_{11}, R_{22} and R_{33} decrease

$\Big\} \; 2l_c > W$

$T_{12}, T_{23}, T_{31} = 1$
$T_{21}, T_{32}, T_{13} = 0$
$R_{11}, R_{22}, R_{33} = 0$

$\Big\} \; 2l_c < W$

$\boxed{3pt}$

e) for $2l_c < W$ μ_2 will copy μ_1

therefore $V = \dfrac{\mu_2 - \mu_1}{eI} = 0$

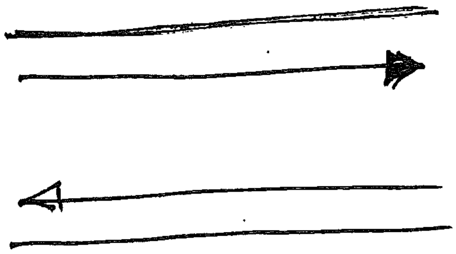
$\boxed{3pt}$

f) In this case μ_2 will copy μ_3
the voltage will increase

$\boxed{3pt}$

C

2(a)

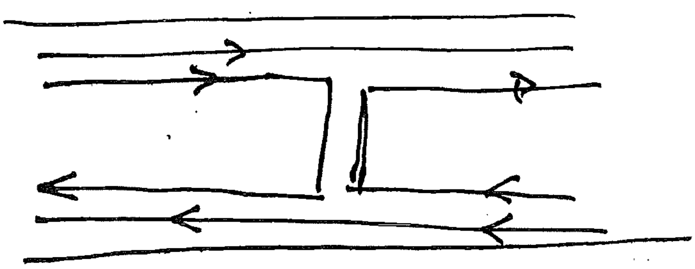


QHE condition.

absence of backscattering when E_F in between Landau levels → edge channel description → R_{xy} quantized and $R_{xx} = 0$

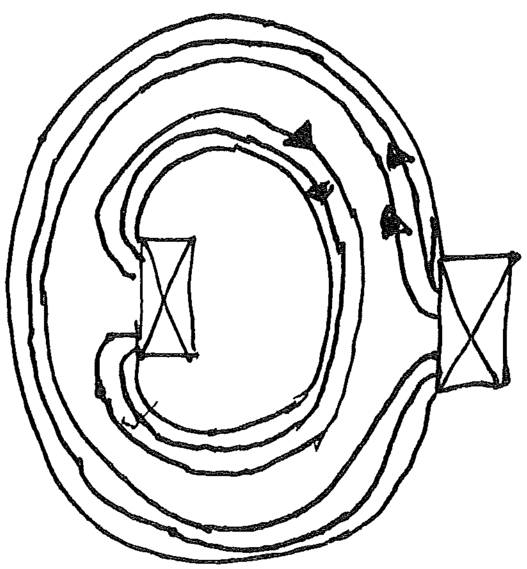
$\boxed{3pt}$

(b)



each LL has a pair of edge channels
total edge channel conductance scales wit N_L
max SdH effect → 1 edge channel reflected
→ effect scales with $\frac{1}{N_L} \sim B$ $\boxed{3pt}$

(c)



two pairs of counter flowing edge channels

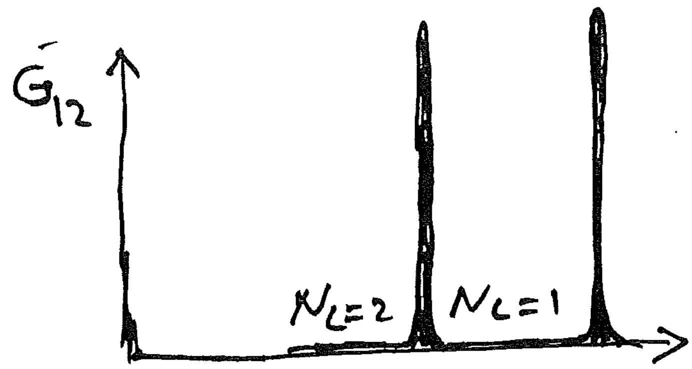
$\boxed{3pt}$

(d) no edge channels connecting inner and outer contact (see (c))

→ no conductance $G_{12} = 0$ $\boxed{3pt}$

(D)

(e)



only conductance G_{12} when there is backscattering between inner and outer edges

$\boxed{3pt}$

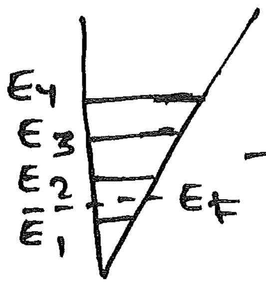
(f) conventional Hall bar: always edge channels present between current carrying contacts

Corbino geometry: no edge channels when E_F is in between consecutive LL. $\boxed{3pt}$

(3) (a)

graphene : 1 atom thick → states are 2D upto next band ($\approx$ 1eV)

2DEG:



→ 2D subbands

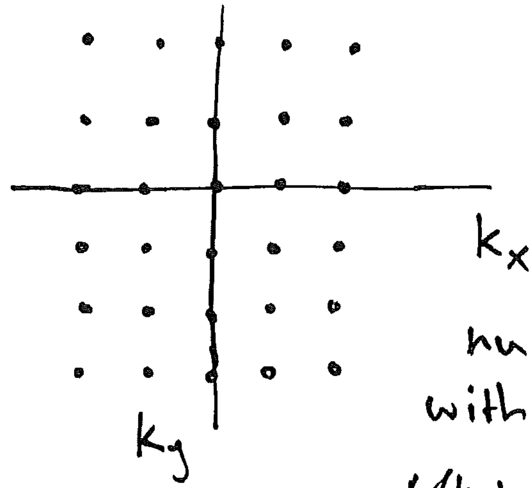
states are 2D for

$E_1 < E_F < E_2$

estimate for $E_1 \sim E_2$ about 10 meV

③ ⓑ

Ⓔ



assume area

$A = L^2 \longrightarrow$ spacing

$\Delta k = \frac{2\pi}{L}$

number of states in sphere with radius k_F =

$$N(k_F) = \frac{2\pi k_F^2}{(2\pi/L)^2} = \frac{L^2 k_F^2}{2\pi}$$

use $E(k_F) = \frac{\hbar^2 k_F^2}{2m^*}$ $\quad k_F^2 = \frac{2m^* E_F}{\hbar^2}$

$$N(k_F) = N(E_F) = \frac{2m^* L^2 E_F}{2\pi\hbar^2} = \frac{m^* L^2 E_F}{\pi\hbar^2}$$

Density of states $= \frac{dN(E_F)}{L^2 dE_F} = \frac{m^*}{\pi\hbar^2}$ (excluding spin)

$\boxed{5pt}$

ⓒ graphene : $N(E_F) = \frac{L^2}{2\pi} \frac{E_F^2}{\hbar^2 v_F^2}$

Density of states : $\frac{dN(E_F)}{dE_F} = \frac{1}{2\pi\hbar^2 v_F^2} \cdot 2E_F = \frac{E_F}{\pi\hbar^2 v_F^2}$

$\boxed{5pt}$

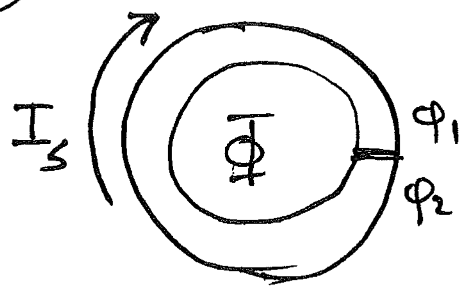
ⓓ differences : graphene has constant Fermi velocity
absence of 180° backscattering

similarities : two dimensional, quantum Hall effect

$\boxed{5pt}$

F

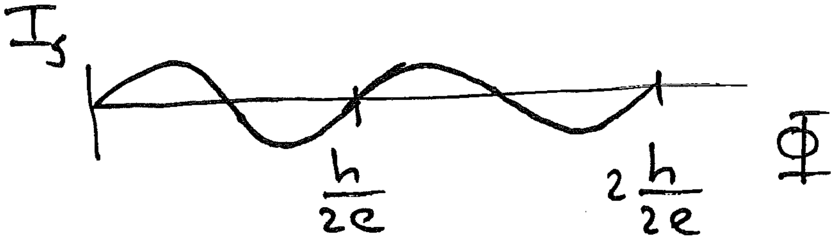
(a)



$I_s = I_c \sin(\varphi_1 - \varphi_2)$

AB effect: $\varphi_1 - \varphi_2 = 2\pi \dfrac{\Phi}{h/2e}$

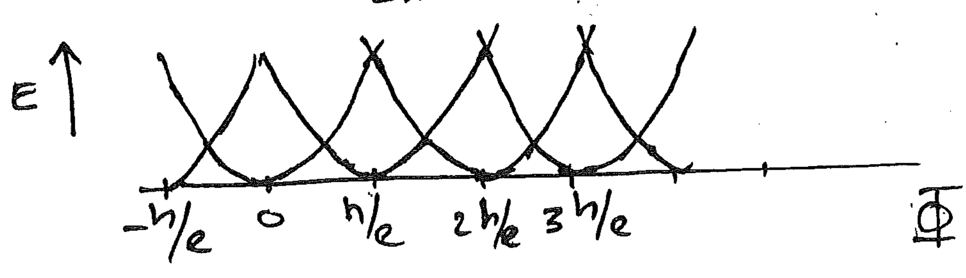
$I_s = I_c \sin\left(\dfrac{\Phi}{h/2e} \cdot 2\pi\right)$ $\quad \Phi = B A$



$\boxed{6\,\mathrm{pt}}$

(b) AB effect : $\Delta\varphi = 2\pi \cdot \dfrac{\Phi}{h/e}$

$E_{kin} = \dfrac{\hbar^2 k^2}{2m^*}$ $\quad k = k_0 + K_{AB}$



$\boxed{6\,\mathrm{pt}}$

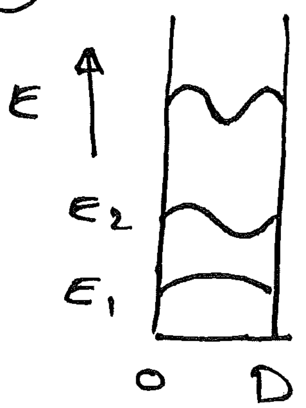
(c) superconductor : max supercurrent $= I_c$
normal conductor : max pers. current $\approx \dfrac{eV_F}{2\pi R}$

superconductor is macroscopic
I does not depent on R $\rightarrow$ decays at T_c
normal metal $I_s \sim \dfrac{1}{R}$ $\longrightarrow$ decays when $kT \sim \Delta E$

$\boxed{4\,\mathrm{pt}}$

5 assume "hard wall" potential (G)

(a)



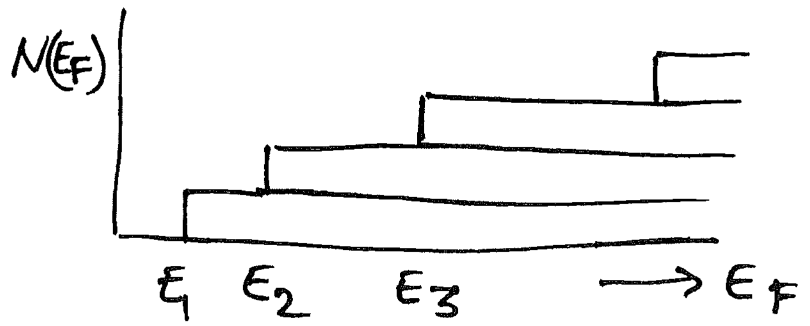
$\lambda = 2D \quad k = \frac{2\pi}{\lambda} = \frac{\pi}{D}$

$E = \frac{\hbar^2 k^2}{2m^*} =$

$E_1 = \frac{\hbar^2}{2m^*}\left(\frac{\pi}{D}\right)^2 \quad E_2 = \frac{\hbar^2}{2m^*}\left(\frac{2\pi}{D}\right)^2$

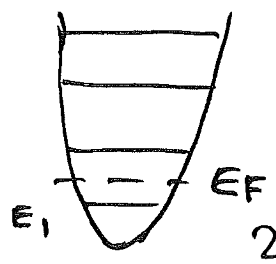
2D when $E_1 < E_F < E_2$ for hard wall potential

(b)



$\boxed{6\,pt}$

(c) harmonic oscillator potential



$E_n = (n + 1/2)\hbar\omega$

2D when $E_1 < E_F < E_2$

$\boxed{6\,pt}$

(d) confinement can matter for:

a) temperature range for which the system is 2D

b) scattering of carriers in potential wel

$\boxed{5\,pt}$